Single Pure – Integration Indefinite

1. Differentiate with respect to x the following:

   (a) $3x^2 + 4x - 1$.

   (b) $3x^2 + 4x$.

   (c) $3x^2 + 4x + 2$.

   (d) $3x^2 + 4x + \frac{3}{2}$.

2. The given expression is for $\frac{dy}{dx}$; integrate to find y. [Don't forget the 'moral of the story' from question 1.]

   (a) x^7. $\boxed{\frac{x^8}{8} + c}$

   (b) $3x^5$. $\boxed{\frac{x^6}{2} + c}$

   (c) $\frac{1}{10}x^4$. $\boxed{\frac{x^5}{50} + c}$

   (d) $\sqrt{x}$. $\boxed{\frac{2x^{3/2}}{3} + c}$

   (e) $\frac{1}{2x^3}$. $\boxed{-\frac{1}{4x^2} + c}$

   (f) $\frac{5}{\sqrt{x}}$. $\boxed{10\sqrt{x} + c}$

   (g) $(2x + 3)^2$. $\boxed{\frac{4}{3}x^3 + 6x^2 + 9x + c}$

   (h) $x^3 + 2x^2 - 3x - 1$. $\boxed{\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 - x + c}$

   (i) $\sqrt{x}\left(x + \frac{1}{x}\right)$. $\boxed{\frac{2}{5}x^{5/2} + 2x^{1/2} + c}$

   (j) $(x + 2)(x^2 - 6)$. $\boxed{\frac{1}{4}x^4 + \frac{2}{3}x^3 - 3x^2 - 12x + c}$

   (k) $\left(3x - \frac{1}{5x}\right)^2$. $\boxed{3x^3 - \frac{6}{5}x - \frac{1}{25x} + c}$

   (l) $4.8x^{-1.4}$. $\boxed{-12x^{-0.4} + c}$

3. Sketch the family of curves that satisfy $\dfrac{dy}{dx} = -x$.

   Find the equation of the member of this family which passes through $(4, 6)$.

4. If $\dfrac{dy}{dx} = 3x^2 - 4x$ and $y = 10$ when $x = 1$, find y in terms of x.

5. The gradient of a curve has the value $x - x^2$ at the point (x, y). If the curve passes through the point $(3, 5)$, find the equation of the curve. Find also the equation of the tangent to the curve at the point $(3, 5)$.

6. Find a formula for y in terms of x if $\dfrac{dy}{dx} = \dfrac{x^2 - 1}{\sqrt{x}}$ and $y = 0$ when $x = 1$.

   Calculate y when $x = 4$. $\boxed{52/5}$

7. The depth of water h metres, in the hold of the *'Slice of life*[1]*'* t hours after she sprang a leak obeyed the law $\frac{dh}{dt} = 1.2t$. How long did it take for the depth to increase to 4 metres.

8. Obtain an expression for V given that $\dfrac{dV}{dx} = (2x^2 - 3)\left(3 + \dfrac{1}{x^2}\right)$.

   If $V = 10$ when $x = 1$, find the value of V when $x = 3$.

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[1]Name the current TV show this is from…

9. It is given that $\frac{dy}{dx}$ is inversely proportional to x^2 and that y and $\frac{dy}{dx}$ are each equal to 1 when $x = 2$. Express y in terms of x.

10. A curve has stationary points at the points where $x = 0$ and $x = 2$. Write down an expression, of the smallest degree possible, for the gradient of the curve. Given that the curve passes through the point $(3, 2)$ and has gradient 4 there, obtain the equation of the curve.

11. A curve has gradient $4x - 3$ at the point (x, y) and the minimum value of y is $-\frac{1}{8}$. Express y in terms of x and sketch the curve.

12. (a) Sketch the graph of $f(x) = \dfrac{1}{x}$.

    (b) Now consider the function $g(x) = \dfrac{1 - \mid x \mid}{x}$.

    Show that
    $$g(x) = \frac{1}{x} - 1 \text{ if } x > 0$$

    and
    $$g(x) = \frac{1}{x} + 1 \text{ if } x < 0.$$

    Hence show that $g'(x) = f'(x)$ for all non-zero values of x.

    (c) Sketch the graph of g, and check that it cannot be obtained by any translation of the graph of f.